JEE Main 2021 August 31, Shift 1 (Mathematics)

1. Find the mean of first 10 numbers of the series $7 \times 8, 10 \times 10, 13 \times 12, 16 \times 14, 19 \times 16 \ldots.$

Ans. 398

Given

$7 \times 8, 10 \times 10, 13 \times 12, 16 \times 14, 19 \times 16 \ldots.$

General term of this series is

$\sum_{r=1}^{n}(3r+4)(2r+6)$

Therefore, $\sum_{r=1}^{n}(3r+4)(2r+6) = \sum_{r=1}^{10}(6r^2+26r+24)$

$$= 6\sum_{r=1}^{10} r^2 + 26\sum_{r=1}^{10} r + \sum_{r=1}^{10} 24$$

$$= 6\left\{\frac{n(n+1)(2n+1)}{6}\right\} + 26\left\{\frac{n(n+1)}{2}\right\} + 24n$$

$$= 6\left\{\frac{10(10+1)(20+1)}{6}\right\} + 26\left\{\frac{10(10+1)}{2}\right\} + 240$$

$$= 6\left\{\frac{10.11.21}{6}\right\} + 26\left\{\frac{10.11}{2}\right\} + 240$$

$$= 2310 + 1430 + 240 = 3980$$

Mean = $\frac{\sum_{r=1}^{10}(3r+4)(2r+6)}{10} = \frac{3980}{10} = 398$

2. Evaluate $\int \frac{dx}{\sqrt[4]{(x-1)^3(x-2)^5}}$.

(A) $-5\left(\frac{x-2}{x-1}\right)^{\frac{1}{5}} + C$

(B) $-5\left(\frac{x-1}{x-2}\right)^{\frac{1}{5}} + C$

(C) $-4\left(\frac{x-2}{x-1}\right)^{\frac{1}{4}} + C$

(D) $-4\left(\frac{x-1}{x-2}\right)^{\frac{1}{4}}+C$

Ans. (D)

$$I=\int \frac{dx}{\sqrt[4]{(x-1)^3(x-2)^5}}$$

$$I=\int \frac{dx}{\left(\frac{(x-2)}{(x-1)}\right)^{\frac{5}{4}}(x-1)^2}$$

Let $\left(\frac{(x-2)}{(x-1)}\right)=t$

$$1-\frac{1}{(x-1)}=t$$

$$\Rightarrow \frac{1}{(x-1)^2}dx=dt$$

$$I=\int \frac{dt}{(\mathrm{t})^{\frac{5}{4}}}$$

$$=\frac{t^{-1/4}}{-\frac{1}{4}}+C$$

$$=\ -4\left(\frac{x-1}{x-2}\right)^{\frac{1}{4}}+C$$

3. If angle between $\vec{a}$ & $\vec{b}$ is $60°$ and $\frac{\vec{a}}{8}$ is a unit vector. If $|2\vec{a}+3\vec{b}|=|3\vec{a}+\vec{b}|$ then the magnitude of $\vec{b}$ is

(A) 5

(B) 7

(C) 10

(D) 13

Ans. (A)

Sol. Given $\frac{\vec{a}}{8}$ is a unit vector.

$$\left|\frac{\vec{a}}{8}\right|=1$$

$\Rightarrow |\vec{a}| = 8$

Also given $|2\vec{a} + 3\vec{b}| = |3\vec{a} + \vec{b}|$

Squaring both sides

$|2\vec{a} + 3\vec{b}|^2 = |3\vec{a} + \vec{b}|^2$

$\Rightarrow 4|\vec{a}|^2 + 9|\vec{b}|^2 + 12|\vec{a}||\vec{b}|cos60° = 9|\vec{a}|^2 + |\vec{b}|^2 + 6|\vec{a}||\vec{b}|cos60°$

$\Rightarrow 5|\vec{a}|^2 = 8|\vec{b}|^2 + 6|\vec{a}||\vec{b}|cos60°$

$\Rightarrow 5(64) = 8|\vec{b}|^2 + 6(8) \cdot |\vec{b}|\frac{1}{2}$

$\Rightarrow |\vec{b}|^2 + 3|\vec{b}| - 40 = 0$

$\Rightarrow (|\vec{b}| + 8)(|\vec{b}| - 5) = 0$

$\Rightarrow |\vec{b}| = 5$ as modulus is always positive.

4. Given a quadratic equation $P(x) = x^2 + ax + 1$. If $P(x)$ is increasing in $[1,2]$ then minimum value of a is A and if $P(x)$ is decreasing in $[1,2]$ then maximum value of a is B then the value of $|A - B|$ is

Ans. 2

Sol. Given

$P(x) = x^2 + ax + 1$ and $x \in [1,2]$

If $P'(x) \geq 0$ then function is increasing in that interval and if $P'(x) \leq 0$ then function is decreasing in that interval.

If $P'(x) = 2x + a \geq 0 \ \forall \ x \in [1,2]$

$\Rightarrow a \geq -2x \ \forall \ x \ \in [1,2]$

$a_{min} = A = -2$

If $P'(x) \leq 0 \ \forall \ x \in [1,2]$

$\Rightarrow 2x + a \leq 0 \ \forall \ x \in [1,2]$

$\Rightarrow a \leq -2x \ \forall \ x \in [1,2]$

$a \leq -4$

$a_{max} = B = -4$

$|A - B| = 2$

5. Find the term independent of x in the expansion $\left(\frac{x}{4}-\frac{12}{x^2}\right)^{12}$

(A) $12_{C_5}\frac{3^5}{4^2}$

(B) $-12_{C_5}\frac{3^5}{4^2}$

(C) $12_{C_6}3^6$

(D) $12_{C_4}\frac{3^4}{4^4}$

Ans. (D)

Sol. Given

$$\left(\frac{x}{4}-\frac{12}{x^2}\right)^{12}$$

General Term of the expansion is

$$T_{r+1}=12_{C_r}\left(\frac{x}{4}\right)^{12-r}\left(-\frac{12}{x^2}\right)^r$$

$$=12_{C_r}\left(\frac{1}{4}\right)^{12-r}(-12)^r\cdot x^{12-r-2r}$$

For independent of x the power of x must be 0.

$\Rightarrow\ 12-3r=0$

$\Rightarrow r=4$

So term independent from x is T_5

$$\Rightarrow T_5=12_{C_4}\cdot\frac{1}{4^8}\cdot 3^4\cdot 4^4$$

$$=12_{C_4}\cdot\frac{3^4}{4^4}$$

6. Find the value of $8\int_{-\frac{1}{2}}^{1}([2x]-|x|)dx$ (where $[\bullet]$ greatest integer function)

(A) -5

(B) 5

(C) 0

(D) $\frac{7}{8}$

Ans. (A)

Sol. To calculate $8\int_{-\frac{1}{2}}^{1}([2x]-|x|)dx$

Let $I=\int_{-\frac{1}{2}}^{1}([2x]-|x|)dx$

Limits of integral to be break where greatest integer function becomes integer

$=\int_{-\frac{1}{2}}^{0}(-1+x)dx+\int_{0}^{\frac{1}{2}}(0-x)dx+\int_{\frac{1}{2}}^{1}(1-x)dx$

$=\left(-x+\frac{x^2}{2}\right)_{-\frac{1}{2}}^{0}-\left(\frac{x^2}{2}\right)_{0}^{\frac{1}{2}}+\left(x-\frac{x^2}{2}\right)_{\frac{1}{2}}^{1}$

$=(0)-\left(\frac{1}{2}+\frac{1}{8}\right)-\frac{1}{8}+\left(1-\frac{1}{2}\right)-\left(\frac{1}{2}-\frac{1}{8}\right)$

$=\ -\frac{5}{8}$

So, $8I=8\int_{-\frac{1}{2}}^{1}([2x]-|x|)dx=-5$

7. Find the sum of first 10 terms of the series $\frac{3}{1^2\times2^2}+\frac{5}{2^2\times3^2}+\frac{7}{3^2\times4^2}+\cdots..$

(A) 1

(B) $\frac{440}{441}$

(C) $\frac{99}{100}$

(D) $\frac{120}{121}$

Ans. (D)

Sol. Given

$\frac{3}{1^2\times2^2}+\frac{5}{2^2\times3^2}+\frac{7}{3^2\times4^2}+\cdots..$

General term of the series

$\sum_{r=1}^{10}\frac{(2r+1)}{r^2\cdot(r+1)^2}$

Using method of difference

$\sum_{r=1}^{10}\frac{(2r+1)}{r^2\cdot(r+1)^2}=\sum_{r=1}^{10}\frac{(r+1)^2-r^2}{r^2\cdot(r+1)^2}$

$= \sum_{r=1}^{10} \left\{\frac{1}{r^2} - \frac{1}{(r+1)^2}\right\}$

$= \left(\frac{1}{1^2} - \frac{1}{2^2}\right) + \left(\frac{1}{2^2} - \frac{1}{3^2}\right) + \left(\frac{1}{3^2} - \frac{1}{4^2}\right) + \cdots .. + \left(\frac{1}{10^2} - \frac{1}{11^2}\right)$

$= 1 - \frac{1}{121} = \frac{120}{121}$

8. Let f be a non-negative function defined on the interval $[0,1]$. If $\int_0^x \sqrt{1-(f'(t))^2}dt = \int_0^x f(t)\text{dt}, 0 \le x \le 1$ and $f(0) = 0$, then the value of $\lim_{x\to 0} \frac{\int_0^x f(t)dt}{x^2}$ is

(A) $-\frac{1}{2}$

(B) $\frac{1}{2}$

(C) 2

(D) 1

Ans. B

$\int_0^x \sqrt{1-(f'(t))^2}dt = \int_0^x f(t)dt$

Differentiate w.r.t. x

$\sqrt{1-(f'(x))^2} = f(x)$

$\Rightarrow 1 - \left(f'(x)\right)^2 = (f(x))^2$

$\Rightarrow \left(f'(x)\right)^2 = 1 - (f(x))^2$

$\Rightarrow \left(\frac{dy}{dx}\right)^2 = 1 - y^2$

$\Rightarrow \frac{dy}{dx} = \pm\sqrt{1-y^2}$

$\Rightarrow \int \frac{dy}{\sqrt{1-y^2}} = \pm \int dx$

by integration

$sin^{-1}y = \pm x + C$

Given $f(0) = 0$

$\because y(0) = 0 \therefore C = 0$

but f is non negative

$\therefore sin^{-1}y = x$

$\Rightarrow y = sinx$

Now,

$\lim_{x\to 0}\frac{\int_0^x f(t)dt}{x^2} = \lim_{x\to 0}\frac{1}{x^2}\int_0^x sintdt$

$= \lim_{x\to 0}\left(\frac{sinx}{2x}\right) = \frac{1}{2}$

9. If a curve follows the differential equation $\frac{dy}{dx} = \frac{2^{x+y}-2^x}{2^y}$ and curve passes through the point $(0,1)$ then find the value of $y(2)$

(A) $y = log_2(e^{-3} + 1)$

(B) $y = log_2(1 + e^3)$

(C) $y = log_2(e^3 - 1)$

(D) $y = log_2(1.e^3)$

Ans. (B)

Sol. Given $\frac{dy}{dx} = \frac{2^{x+y}-2^x}{2^y}$

$\Rightarrow \frac{dy}{dx} = \frac{2^{x+y}-2^x}{2^y} = 2^x\left(\frac{2^y-1}{2^y}\right)$

Applying variable separable form

$\int \frac{2^y dy}{2^y - 1} = \int 2^x dx$

Let $2^y - 1 = t$

$\Rightarrow 2^y ln2dy = dt$

$\Rightarrow \frac{1}{ln2}\int \frac{dt}{t} = \int 2^x dx$

$\Rightarrow \frac{lnt}{ln2} = \frac{2^x}{ln2} + \frac{c}{ln2}$

$lnt = 2^x + C$

$\Rightarrow ln(2^y - 1) = 2^x + C$

when $x=0, y=1, C=-1$

$ln(2^y-1)=2^x-1$

$\Rightarrow 2^y-1=e^{2^x-1}$

$\Rightarrow 2^y=1+e^{2^x-1}$

When $x=2$

$\Rightarrow 2^y=1+e^3$

$\Rightarrow y=log_2(1+e^3)$

10. Distance of lines $xcosec\theta+ysec\theta=kcot2\theta$ and $xsin\theta+ycos\theta=ksin2\theta$ from origin is p and q respectively, then which of the following is correct

(A) $4q^2+p^2=4k^2$

(B) $p^2+q^2=4k^2$

(C) $4p^2+q^2=k^2$

(D) $4p^2+q^2=k^2$

Ans. (C)

Given

$L_1: \ xcosec\theta+ysec\theta=kcot2\theta$

$$\frac{x}{sin\theta}+\frac{y}{cos\theta}=k\frac{cos2\theta}{sin2\theta}$$

$$xcos\theta+ysin\theta=\frac{k}{2}cos2\theta$$

Given that Perpendicular distance from origin$(0,0)$ is p

$$\Rightarrow p=\left|\frac{0+0-\frac{k}{2}cos2\theta}{\sqrt{cos^2\theta+sin^2\theta}}\right|$$

$$\Rightarrow p^2=\frac{k^2}{4}cos^2 2\theta.....(i)$$

$L_2: xsin\theta+ycos\theta-ksin2\theta=0$

Given that Perpendicular distance from origin$(0,0)$ is q

$$\Rightarrow q=\left|\frac{0+0-ksin2\theta}{\sqrt{sin^2\theta+cos^2\theta}}\right|$$

$\Rightarrow q^2 = k^2 sin^2 2\theta \ldots .(ii)$

From equation (i) & (ii)

$4p^2 + q^2 = k^2(sin^2 2\theta + cos^2 2\theta)$

$\Rightarrow 4p^2 + q^2 = k^2$

11. If $a_r = \cos\left(\frac{2\pi r}{9}\right) + i\sin\left(\frac{2\pi r}{9}\right), (i = \sqrt{-1}), r = 1,2, \ldots \ldots ,$ then the value of $\begin{vmatrix} a_1 & a_2 & a_3 \\ a_4 & a_5 & a_6 \\ a_7 & a_8 & a_9 \end{vmatrix}$ is

(A) $a_4 a_9 - a_7 a_6$

(B) a_9

(C) a_1

(D) $a_3 a_8 - a_7 a_5$

Ans. (A)

Sol.

Given $a_r = \cos\left(\frac{2\pi r}{9}\right) + i\sin\left(\frac{2\pi r}{9}\right), (i = \sqrt{-1}), r = 1,2 \ldots \ldots$

Converting it into Euler's form

$a_r = e^{i\left(\frac{2\pi r}{9}\right)}$

Now, $\begin{vmatrix} a_1 & a_2 & a_3 \\ a_4 & a_5 & a_6 \\ a_7 & a_8 & a_9 \end{vmatrix}$

$$= \begin{vmatrix} e^{i\frac{2\pi}{9}} & e^{\frac{4\pi}{9}} & e^{\frac{6\pi}{9}} \\ e^{\frac{8\pi}{9}} & e^{i\frac{10\pi}{9}} & e^{\frac{12\pi}{9}} \\ e^{\frac{14\pi}{9}} & e^{\frac{16\pi}{9}} & e^{\frac{18\pi}{9}} \end{vmatrix}$$

$$= e^{i\left(\frac{2\pi}{9}+\frac{8\pi}{9}+\frac{14\pi}{9}\right)} \begin{vmatrix} 1 & e^{i\frac{2\pi}{9}} & e^{i\frac{4\pi}{9}} \\ 1 & e^{i\frac{2\pi}{9}} & e^{i\frac{4\pi}{9}} \\ 1 & e^{i\frac{2\pi}{9}} & e^{i\frac{4\pi}{9}} \end{vmatrix} = 0$$

Now $a_r = e^{i\left(\frac{2\pi r}{9}\right)}$

Checking all the options

$a_4 a_9 - a_7 a_6 = e^{i\left(\frac{8\pi}{9}+\frac{18\pi}{9}\right)} - e^{i\left(\frac{14\pi}{9}+\frac{12\pi}{9}\right)} = 0,$

$a_9 = e^{i(2\pi)} = 1,$

$a_1 = e^{i\frac{2\pi}{9}},$

$a_3 a_8 - a_7 a_5 = e^{i\left(\frac{6\pi}{9}+\frac{16\pi}{9}\right)} - e^{i\left(\frac{14\pi}{9}+\frac{10\pi}{9}\right)} \neq 0$

12. Three terms form an increasing G.P. with common ratio r.If the second term of the given G.P. is doubled then the new series is in A.P. with common difference d also the 4^{th} term of G.P. is $3r^2$. Then the value of $(r^2 - d)$ is:

(A) $7 + 3\sqrt{3}$

(B) $7 + \sqrt{3}$

(C) $7 - \sqrt{3}$

(D) $7 - 3\sqrt{3}$

Ans. (B)

Sol. Three terms form an increasing G.P.

Let three terms are $\frac{a}{r}, a, ar$ and also given 4^{th} term of G.P. is $3r^2$.

$\frac{a}{r}, a, ar, 3r^2 \rightarrow G.P.\ldots..(i)$

After doubling second term

$\frac{a}{r}, 2a, ar \rightarrow A.P.\ldots..(ii)$

From equation (i)

Using properties of G.P.

$a^2 r = 3ar$ as $r \neq 0$ and $a \neq 0$ for increasing G.P.

$\Rightarrow a = 3$

From equation (ii)

Using properties of A.P.

$4a = ar + \frac{a}{r}$

$4 = r + \frac{1}{r}$

$r^2 + 1 = 4r$

$r^2 - 4r + 1 = 0$

$\Rightarrow r = \dfrac{4 \pm \sqrt{12}}{2} = 2 + \sqrt{3}, \qquad 2 - \sqrt{3} \quad ((2 - \sqrt{3})\ rejected\ )$

$d = 2a - \frac{a}{r}$

$= a\left(2 - \dfrac{1}{r}\right)$

$= 3\left(2 - \dfrac{1}{2 + \sqrt{3}}\right)$

$= 3(2 - 2 + \sqrt{3}) = 3\sqrt{3}$

Hence, $r^2 - d = (2 + \sqrt{3})^2 - (3\sqrt{3}) = 7 + \sqrt{3}$

13. The value of $\lim\limits_{x \to 0} \frac{\sin^2(\pi \cos^4 x)}{x^4}$ is equal to

(A) 4π

(B) π^2

(C) $2\pi^2$

(D) $4\pi^2$

Ans. (D)

Sol. $\lim\limits_{x \to 0} \frac{\sin^2\left(\pi(1 - \sin^2 x)^2\right)}{x^4}$

$= \lim\limits_{x \to 0} \frac{\sin^2\left(\pi(1 + \sin^4 x - 2\sin^2 x)^2\right)}{x^4}$

$= \lim\limits_{x \to 0} \frac{\sin^2\left(\pi - \pi(2\sin^2 x - \sin^4 x)\right)}{x^4}$

$= \lim\limits_{x \to 0} \frac{\mathrm{sir}^2\left(\pi(2\sin^2 x - \sin^4 x)\right)}{x^4}$

$= \lim\limits_{x \to 0} \left(\frac{\sin[\pi(2\sin^2 x - \sin^4 x)]}{\pi(2\sin^2 x - \sin^4 x)}\right)^2 \cdot \frac{\pi^2(2\sin^2 x - \sin^4 x)^2}{x^4}$

$= \pi^2 \lim\limits_{x \to 0} \left(\frac{\sin x}{x}\right)^4 \cdot (2 - \sin^2 x)^2$

Substituting the limits and using standard formula of limits

$$\lim_{x\to 0}\frac{\sin^2(\pi(1-\sin^2\ x)^2)}{x^4}=4\pi^2$$

14. The number of words made by the alphabets of 'VOWELS' such that consonants are not together is:

Ans. 0

Sol. Given word is 'VOWELS' in which number of consonants are four 'VWLS' and number of vowels are two 'OE'.

As per asked condition when no consonants are together is not possible to form any number of words.

15. If vertex and focus of a parabola drawn on the side of positive x axis are $(R, 0)$ and $(S, 0)$ respectively then the length of latus rectum is

(A) $2(S-R)$

(B) $S-R$

(C) $4(S-R)$

(D) $4S$

Ans. (C)

Sol. $y^2=4a(x-R)$

$y^2=4(S-R)(x-R)$

Length of $LR=4(S-R)$

16. The quadratic equation having $\operatorname{cosec}(18^\circ)$ as its root, is

(A) $x^2-2x-4=0$

(B) $x^2-2x+4=0$

(C) $x^2-2x-1=0$

(D) $x^2+2x-1=0$

Ans. (A)

Sol.

$$\operatorname{cosec}18^\circ=\frac{4}{\sqrt{5}-1}\times\frac{\sqrt{5}+1}{\sqrt{5}+1}=\sqrt{5}+1$$

Hence equation $x=\sqrt{5}+1$

$(x-1)^2=5$

$x^2 - 2x - 4 = 0$

17. If the statement $(p * \sim q) \rightarrow (p \square q)$ is a tautology, then

(A) $*$ is $\wedge$, $\square$ is $\wedge$

(B) $*$ is $\vee$, $\square$ is $\wedge$

(C) $*$ is $\wedge$, $\square$ is $\vee$

(D) $*$ is $\vee$, $\square$ is $\vee$

Ans. (C)

Sol. $(p \wedge \sim q) \rightarrow (p \vee q)$

$\sim (p \wedge \sim q) \vee (p \vee q)$

$(\sim p \vee q) \vee (p \vee q)$

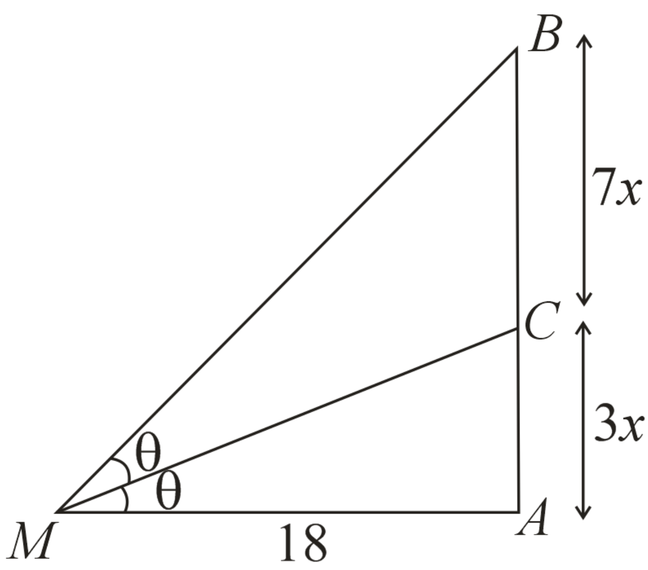
$= (\sim p \vee p) \vee (q \vee q) = t \vee q$

$= t$

18. A pole AB is $18cm$ apart from a point M. Point C lies in the pole AB such that $\frac{AC}{AB} = \frac{3}{10}$ and the angle of elevation of a point B is twice the angle of elevation of point C for a man standing at point M. If the height of the pole is $12\sqrt{k}\ cm$, then the value of k is equal to

Ans. 10

Sol.



In the triangle ΔACM, $tan\theta = \frac{3x}{18} = \frac{x}{6}$

In the triangle ΔABM, $tan2\theta = \frac{10x}{18} = \frac{5x}{9}$

Now, using $tan2\theta = \frac{2tan\theta}{1-tan^2\theta}$, we get

$\Rightarrow \frac{5x}{9} = \frac{2\frac{x}{6}}{1-\frac{x^2}{36}}$

$\Rightarrow 5 = \frac{3}{1-\frac{x^2}{36}}$

$\Rightarrow 1 - \frac{x^2}{36} = \frac{3}{5}$

$\Rightarrow \frac{x^2}{36} = \frac{2}{5}$

$\Rightarrow x^2 = \frac{72}{5} \Rightarrow x = \frac{6\sqrt{2}}{\sqrt{5}} cm$

$\Rightarrow$ Length of pole $= 10x = \frac{60\sqrt{2}}{\sqrt{5}} = \frac{60\sqrt{10}}{5} = 12\sqrt{10}\ cm$

19. If the function $f(x) = \begin{cases} \frac{\log\left(\frac{1+\frac{x}{b}}{1-\frac{x}{a}}\right)}{x} & x < 0 \\ k & x = 0 \\ \frac{\cos^2 x - \sin^2 x - 1}{\sqrt{x^2+1}-1} & x > 0 \end{cases}$ is continuous at $x = 0$, then the absolute value of $\frac{1}{a} + \frac{1}{b} + \frac{4}{k}$ is equal to

Ans. 5

Sol.

$$\text{LHL} = \lim_{x\to 0^-} \frac{\log\left(\frac{1+\frac{x}{b}}{1-\frac{x}{a}}\right)}{x}$$

$$= \lim_{x\to 0^-} \frac{\log\left(1+\frac{x}{b}\right)-\log\left(1-\frac{x}{a}\right)}{x}$$

$$= \lim_{x\to 0^-} \frac{\log\left(1+\frac{x}{b}\right)}{\frac{x}{b}} \times \frac{1}{b} - \frac{\log\left(1-\frac{x}{a}\right)}{\frac{x}{a}} \times \frac{1}{a} = \frac{1}{b} + \frac{1}{a}$$

$$\text{RHL} = \lim_{x\to 0^+} \frac{\cos^2 x - \sin^2 x - 1}{\sqrt{x^2+1}-1} \times \frac{\sqrt{x^2+1}+1}{\sqrt{x^2+1}+1}$$

$$= \lim_{x\to 0^+} \frac{-2\sin^2 x}{x^2}\left(\sqrt{x^2+1}+1\right) = -4$$

So, $\frac{1}{b} + \frac{1}{a} = k = -4 \Rightarrow \frac{1}{a} + \frac{1}{b} + \frac{4}{k} = -4 - 1 = -5$

20. If the system of equations $2x + y + z = 3, x - y + z = -1$ and $x + y + az = b$ has no solution, then

(A) $a \neq \frac{1}{3}, b = \frac{7}{3}$

(B) $a = \frac{1}{3}, b = \frac{7}{3}$

(C) $\text{a} = \frac{1}{3}, \text{b} \neq \frac{7}{3}$

(D) $a \neq \frac{1}{3}, b \neq \frac{7}{3}$

Ans. (C)

Sol. For no solution, the condition is $D = 0$ and not all of D_1, D_2, D_3 is equal to 0

$$D = \begin{vmatrix} 2 & 1 & 1 \\ 1 & -1 & 1 \\ 1 & 1 & a \end{vmatrix} = 0 \Rightarrow 2 - 1 - 3a = 0 \Rightarrow a = \frac{1}{3}$$

$$D_1 = \begin{vmatrix} 3 & 1 & 1 \\ -1 & -1 & 1 \\ b & 1 & a \end{vmatrix} \Rightarrow 2\left(b - \frac{7}{3}\right)$$

$$D_2 = \begin{vmatrix} 2 & 3 & 1 \\ 1 & -1 & 1 \\ 1 & b & a \end{vmatrix} \Rightarrow \frac{7}{3} - b$$

$$D_3 = \begin{vmatrix} 2 & 1 & 3 \\ 1 & -1 & -1 \\ 1 & 1 & b \end{vmatrix} \Rightarrow 7 - 3b$$

For the system to have no solution

$\Rightarrow b \neq \frac{7}{3}$