FINAL JEE–MAIN EXAMINATION – JULY, 2021

(Held On Tuesday 20th July, 2021) **TIME : 9 : 00 AM to 12 : 00 NOON**

MATHEMATICS — TEST PAPER WITH ANSWER

SECTION-A

1. The Boolean expression $(p \wedge \sim q) \Rightarrow (q \vee \sim p)$ is equivalent to :

(1) $q \Rightarrow p$ (2) $p \Rightarrow q$

(3) $\sim q \Rightarrow p$ (4) $p \Rightarrow \sim q$

Official Ans. by NTA (2)

2. Let a be a positive real number such that $\int_0^a e^{x-[x]}\,dx = 10e - 9$ where [x] is the greatest integer less than or equal to x. Then a is equal to :

(1) $10 - \log_e(1 + e)$ (2) $10 + \log_e 2$

(3) $10 + \log_e 3$ (4) $10 + \log_e(1+ e)$

Official Ans. by NTA (2)

3. The mean of 6 distinct observations is 6.5 and their variance is 10.25. If 4 out of 6 observations are 2, 4, 5 and 7, then the remaining two observations are:

(1) 10, 11 (2) 3, 18

(3) 8, 13 (4) 1, 20

Official Ans. by NTA (1)

4. The value of the integral $\int_{-1}^{1} \log_e(\sqrt{1-x} + \sqrt{1+x})dx$ is equal to :

(1) $\frac{1}{2}\log_e 2 + \frac{\pi}{4} - \frac{3}{2}$ (2) $2\log_e 2 + \frac{\pi}{4} - 1$

(3) $\log_e 2 + \frac{\pi}{2} - 1$ (4) $2\log_e 2 + \frac{\pi}{2} - \frac{1}{2}$

Official Ans. by NTA (2)

ALLEN Ans. (3)

5. If α and β are the distinct roots of the equation $x^2 + (3)^{1/4}x + 3^{1/2} = 0$, then the value of $\alpha^{96}(\alpha^{12} - 1) + \beta^{96}(\beta^{12} - 1)$ is equal to :

(1) 56×3^{25} (2) 56×3^{24}

(3) 52×3^{24} (4) 28×3^{25}

Official Ans. by NTA (3)

6. Let $A = \begin{bmatrix} 2 & 3 \\ a & 0 \end{bmatrix}$, $a \in \mathbf{R}$ be written as P + Q where P is a symmetric matrix and Q is skew symmetric matrix. If det(Q) = 9, then the modulus of the sum of all possible values of determinant of P is equal to :

(1) 36 (2) 24 (3) 45 (4) 18

Official Ans. by NTA (1)

7. If z and ω are two complex numbers such that $|z\omega| = 1$ and $\arg(z) - \arg(\omega) = \frac{3\pi}{2}$, then $\arg\left(\frac{1-2\bar{z}\omega}{1+3\bar{z}\omega}\right)$ is :

(Here arg(z) denotes the principal argument of complex number z)

(1) $\frac{\pi}{4}$ (2) $-\frac{3\pi}{4}$ (3) $-\frac{\pi}{4}$ (4) $\frac{3\pi}{4}$

Official Ans. by NTA (3)

ALLEN Ans. (2)

8. If in a triangle ABC, AB = 5 units, $\angle B = \cos^{-1}\left(\frac{3}{5}\right)$ and radius of circumcircle of ΔABC is 5 units, then the area (in sq. units) of ΔABC is :

(1) $10 + 6\sqrt{2}$ (2) $8 + 2\sqrt{2}$

(3) $6 + 8\sqrt{3}$ (4) $4 + 2\sqrt{3}$

Official Ans. by NTA (3)

9. Let [x] denote the greatest integer ≤ x, where $x \in \mathbf{R}$. If the domain of the real valued function

$$f(x) = \sqrt{\frac{|[x]| - 2}{|[x]| - 3}}$$

is $(-\infty, a) \cup [b, c) \cup [4, \infty), a < b < c$, then the value of a + b + c is:

(1) 8 (2) 1

(3) –2 (4) –3

Official Ans. by NTA (3)

10. Let y = y(x) be the solution of the differential equation $x\tan\left(\frac{y}{x}\right)dy = \left(y\tan\left(\frac{y}{x}\right) - x\right)dx$, $-1 \le x \le 1, y\left(\frac{1}{2}\right) = \frac{\pi}{6}$. Then the area of the region bounded by the curves $x = 0$, $x = \frac{1}{\sqrt{2}}$ and $y = y(x)$ in the upper half plane is:

(1) $\frac{1}{8}(\pi - 1)$ (2) $\frac{1}{12}(\pi - 3)$

(3) $\frac{1}{4}(\pi - 2)$ (4) $\frac{1}{6}(\pi - 1)$

Official Ans. by NTA (1)

11. The coefficient of x^{256} in the expansion of $(1-x)^{101}(x^2+x+1)^{100}$ is:

(1) ${}^{100}C_{16}$ (2) ${}^{100}C_{15}$

(3) $-{}^{100}C_{16}$ (4) $-{}^{100}C_{15}$

Official Ans. by NTA (2)

12. Let $A = [a_{ij}]$ be a 3×3 matrix, where

$$a_{ij} = \begin{cases} 1 & , \quad \text{if } i = j \\ -x & , \quad \text{if } |i-j| = 1 \\ 2x+1 & , \quad \text{otherwise.} \end{cases}$$

Let a function $f : \mathbf{R} \to \mathbf{R}$ be defined as $f(x) = \det(A)$. Then the sum of maximum and minimum values of f on $\mathbf{R}$ is equal to:

(1) $-\frac{20}{27}$ (2) $\frac{88}{27}$

(3) $\frac{20}{27}$ (4) $-\frac{88}{27}$

Official Ans. by NTA (4)

13. Let $\vec{a} = 2\hat{i} + \hat{j} - 2\hat{k}$ and $\vec{b} = \hat{i} + \hat{j}$. If $\vec{c}$ is a vector such that $\vec{a}\cdot\vec{c} = |\vec{c}|, |\vec{c} - \vec{a}| = 2\sqrt{2}$ and the angle between $(\vec{a}\times\vec{b})$ and $\vec{c}$ is $\frac{\pi}{6}$, then the value of $\left|(\vec{a}\times\vec{b})\times\vec{c}\right|$ is :

(1) $\frac{2}{3}$ (2) 4

(3) 3 (4) $\frac{3}{2}$

Official Ans. by NTA (4)

14. The number of real roots of the equation $\tan^{-1}\sqrt{x(x+1)} + \sin^{-1}\sqrt{x^2+x+1} = \frac{\pi}{4}$ is :

(1) 1 (2) 2

(3) 4 (4) 0

Official Ans. by NTA (4)

15. Let $y = y(x)$ be the solution of the differential equation $e^x\sqrt{1-y^2}\,dx + \left(\frac{y}{x}\right)dy = 0, y(1) = -1.$

Then the value of $(y(3))^2$ is equal to:

(1) $1 - 4e^3$ (2) $1 - 4e^6$

(3) $1 + 4e^3$ (4) $1 + 4e^6$

Official Ans. by NTA (2)

16. Let 'a' be a real number such that the function $f(x) = ax^2 + 6x - 15$, $x \in \mathbf{R}$ is increasing in $\left(-\infty, \frac{3}{4}\right)$ and decreasing in $\left(\frac{3}{4}, \infty\right)$. Then the function $g(x) = ax^2 - 6x + 15$, $x \in \mathbf{R}$ has a:

(1) local maximum at $x = -\frac{3}{4}$

(2) local minimum at $x = -\frac{3}{4}$

(3) local maximum at $x = \frac{3}{4}$

(4) local minimum at $x = \frac{3}{4}$

Official Ans. by NTA (1)

17. Let a function $f : \mathbf{R} \to \mathbf{R}$ be defined as

$$f(x) = \begin{cases} \sin x - e^x & \text{if } x \le 0 \\ a + [-x] & \text{if } 0 < x < 1 \\ 2x - b & \text{if } x \ge 1 \end{cases}$$

Where [x] is the greatest integer less than or equal to x. If f is continuous on $\mathbf{R}$, then (a + b) is equal to:

(1) 4 (2) 3

(3) 2 (4) 5

Official Ans. by NTA (2)

18. Words with or without meaning are to be formed using all the letters of the word EXAMINATION. The probability that the letter M appears at the fourth position in any such word is:

(1) $\frac{1}{66}$ (2) $\frac{1}{11}$ (3) $\frac{1}{9}$ (4) $\frac{2}{11}$

Official Ans. by NTA (2)

19. The probability of selecting integers $a \in [-5, 30]$ such that $x^2 + 2(a+4)x - 5a + 64 > 0$, for all $x \in \mathbf{R}$, is:

(1) $\frac{7}{36}$ (2) $\frac{2}{9}$ (3) $\frac{1}{6}$ (4) $\frac{1}{4}$

Official Ans. by NTA (2)

20. Let the tangent to the parabola $S : y^2 = 2x$ at the point P(2, 2) meet the x-axis at Q and normal at it meet the parabola S at the point R. Then the area (in sq. units) of the triangle PQR is equal to:

(1) $\frac{25}{2}$ (2) $\frac{35}{2}$ (3) $\frac{15}{2}$ (4) 25

Official Ans. by NTA (1)

SECTION-B

1. Let $\vec{a}$, $\vec{b}$, $\vec{c}$ be three mutually perpendicular vectors of the same magnitude and equally inclined at an angle θ, with the vector $\vec{a}+\vec{b}+\vec{c}$. Then $36\cos^2 2\theta$ is equal to ________.

Official Ans. by NTA (4)

2. Let $A=\begin{pmatrix}1 & -1 & 0\\ 0 & 1 & -1\\ 0 & 0 & 1\end{pmatrix}$ and $B = 7A^{20} - 20A^7 + 2I$, where I is an identity matrix of order 3×3. If $B = [b_{ij}]$, then b_{13} is equal to ________.

Official Ans. by NTA (910)

3. Let P be a plane passing through the points (1, 0, 1), (1, –2, 1) and (0, 1, –2). Let a vector $\vec{a}=\alpha\hat{i}+\beta\hat{j}+\gamma\hat{k}$ be such that $\vec{a}$ is parallel to the plane P, perpendicular to $\left(\hat{i}+2\hat{j}+3\hat{k}\right)$ and $\vec{a}\cdot\left(\hat{i}+\hat{j}+2\hat{k}\right)=2$, then $(\alpha - \beta + \gamma)^2$ equals ________.

Official Ans. by NTA (81)

4. The number of rational terms in the binomial expansion of $\left(4^{\frac{1}{4}}+5^{\frac{1}{6}}\right)^{120}$ is ________.

Official Ans. by NTA (21)

5. If the shortest distance between the lines $\vec{r_1}=\alpha\hat{i}+2\hat{j}+2\hat{k}+\lambda\left(\hat{i}-2\hat{j}+2\hat{k}\right)$, $\lambda \in \mathbf{R}$, $\alpha > 0$ and $\vec{r_2}=-4\hat{i}-\hat{k}+\mu\left(3\hat{i}-2\hat{j}-2\hat{k}\right)$, $\mu \in \mathbf{R}$ is 9, then α is equal to ________.

Official Ans. by NTA (6)

6. Let T be the tangent to the ellipse $E : x^2 + 4y^2 = 5$ at the point P(1, 1). If the area of the region bounded by the tangent T, ellipse E, lines $x = 1$ and $x = \sqrt{5}$ is $\alpha\sqrt{5} + \beta + \gamma\cos^{-1}\left(\frac{1}{\sqrt{5}}\right)$, then $|\alpha+\beta+\gamma|$ is equal to ________.

Official Ans. by NTA (1)

7. Let a, b, c, d be in arithmetic progression with common difference λ. If

$$\begin{vmatrix}x+a-c & x+b & x+a\\ x-1 & x+c & x+b\\ x-b+d & x+d & x+c\end{vmatrix}=2,$$

then value of λ^2 is equal to ________.

Official Ans. by NTA (1)

8. There are 15 players in a cricket team, out of which 6 are bowlers, 7 are batsmen and 2 are wicketkeepers. The number of ways, a team of 11 players be selected from them so as to include at least 4 bowlers, 5 batsmen and 1 wicketkeeper, is________.

Official Ans. by NTA (777)

9. Let $y = mx + c$, $m > 0$ be the focal chord of $y^2 = -64x$, which is tangent to $(x + 10)^2 + y^2 = 4$. Then, the value of $4\sqrt{2}(m + c)$ is equal to________.

Official Ans. by NTA (34)

10. If the value of $\lim\limits_{x\to 0}\left(2-\cos x\sqrt{\cos 2x}\right)^{\left(\frac{x+2}{x^2}\right)}$ is equal to e^a, then a is equal to ________.

Official Ans. by NTA (3)